Supplementary Methods

This document describes the models in
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Circuit-based bistability model

The circuit-based bistability network uses 300 two-compartment neurons divided into 100 units, containing 3 neurons each. We capture the spatial extent of the neuronal morphology using a two-compartment model (Pinsky and Rinzel, 1994). We represent the soma and the axon lumped into one compartment containing the currents necessary for spike generation (I_{Na} and I_{KDr}). The dendritic compartment contains only the leak and synaptic currents. The membrane voltage of the ith neuron obeys the current balance equations:

$$C_m \frac{dV_i^{\mathrm{s}}}{dt} = -I^{\mathrm{s}}_{Leak,i} - I_{Na,i} - I_{KDr,i} - \frac{g_c}{p}(V_i^{\mathrm{s}} - V_i^{\mathrm{d}}) \tag{1}$$

$$C_m \frac{dV_i^{\mathrm{d}}}{dt} = -I^{\mathrm{d}}_{Leak,i} - \frac{g_c}{1-p}(V_i^{\mathrm{d}} - V_i^{\mathrm{s}}) - I_{syn}, \tag{2}$$

where C_m is the specific membrane capacitance which is taken to be 0.5 μF/cm^2 for both the dendrite and the soma for all cells, p is the ratio of the somatic membrane area to the total area and g_c is the equivalent axial conductance between the two lumped compartments. These latter parameters (p =0.5, g_c =0.5 mS/cm^2) determine the electrotonic structure of the neuron.

The passive leak current in both the soma and dendrites were modelled as $I_{Leak} = g_{leak}(V - E_{leak})$, where g_{leak} was the leak conductance which was taken to be 0.1 mS/cm^2 for the soma and dendrite. E_{leak} = -80mV was the leak reversal potential for both the compartments. The voltage-dependent currents were modelled according to the Hodgkin-Huxley formalism, with the gating variables obeying the equation:

$$\frac{dx}{dt} = \phi_x(\alpha_x(V)(1-x) - \beta_x(V)x) = \phi_x\Big(\frac{x_\infty(V) - x}{\tau_x(V)}\Big), \qquad (3)$$

where x represents the activation/inactivation gates for the voltage-dependent currents.

The sodium current, $I_{Na} = g_{Na}m^3h(V^{\mathrm{s}} - E_{Na})$, where $g_{Na} = 45$ mS/cm^2 and sodium reversal potential, $E_{Na} = 55$ mV with $\alpha_m(V) = -0.1(V + 32)/[\exp(-(V+32)/10) - 1]$, $\beta_m(V) = 4\exp(-[V+57]/18)$;

$\alpha_h(V) = 0.07\exp(-[V+48]/20)$ and $\beta_h(V) = 1/[\exp(-\{V+18\}/10)+1]$, with $\phi_m = \phi_h = 2.5$. The delayed rectifier potassium current, $I_{KDr} = g_K n^4(V^{\mathrm{s}} - E_K)$, where $g_K = 18$ mS/cm^2 and potassium reversal potential, $E_K = -80$ mV with $\alpha_n(V) = -0.01(V+34)/[\exp(-(V+34)/10) - 1]$, $\beta_n(V) = 0.125\exp(-[V+44]/80)$, with $\phi_n = 2.5$.

The synaptic current into each neuron is due to the recurrent connections and the external (vestibular) inputs:

$$I_{syn,i} = I_{NMDA,i} + I_{ext}, \qquad (4)$$

with

$$I_{NMDA,i} = g_{NMDA,i}\sum_j s_j(V_i^{\mathrm{d}} - E_{syn})/(1 + 0.3[\mathrm{Mg}^{2+}]\exp(-0.08V_i^{\mathrm{d}})), \qquad (5)$$

where the magnesium concentration, $[\mathrm{Mg}^{2+}] = 0.5$mM and $g_{NMDA,i} = 22\mu$S/cm^2 for synapses connecting neurons in the same unit and $g_{NMDA,i} = g_2$ for inter-unit connections. $g_2 = 1.3$, 1.2, 1.4 and 1.3 $\mu S/\mathrm{cm}^2$ for figures 5C,D,E and 6 respectively. The summation index j is over all the presynaptic neurons in the network. The NMDA gating variable obeys second-order kinetics:

$$\frac{dx_j}{dt} = F(V_j^{\mathrm{s}})(1 - x_j) - 0.5x_j \tag{6}$$

$$\frac{ds_j}{dt} = 0.5x_j(1 - s_j) - 0.01s_j. \tag{7}$$

A bias current is applied to all neurons. The bias current is the same for the neurons belonging to the same unit. Across units, it is uniformly (spaced/distributed) in the interval (0,1.5)μA/cm^2. The total input current into the neuron receiving the maximum current in the whole population is shown in Figures 5 and 6. Current pulses representing burst inputs were injected into the dendrites as shown in Figure 6.

NMDAR-based bistability model

The NMDAR-mediated network bistability model uses a recurrent, all-to-all connected network of spiking neurons. We use a two-compartment neuron model similar to the circuit-based implementation above, with slight modifications *(A. Kepecs and S. Raghavachari, "3-state neurons for contextual processing", Advances in Neural Information Processing Systems 14, edited by T. G. Dietterich and S. Becker and Z. Ghahramani, (MIT Press, Cambridge 2002)).* The somatic compartment has leak and spiking currents, while the dendritic compartment contains additional voltage-dependent currents (I_{NaP} and two potassium currents, I_{Ks} and I_{KA}) and receives recurrent connections from the network via AMPA and NMDA synapses as well as external inputs. Thus, the somatic voltage, V_i^{s} of a single neuron obeys the current balance equation:

$$C_m \frac{dV_i^{\mathrm{s}}}{dt} = -I_{leak,i}^{\mathrm{s}} - I_{Na,i} - I_{KDr,i} - \frac{g_c}{p}(V_i^{\mathrm{s}} - V_i^{\mathrm{d}}) \tag{8}$$

while the dendritic voltage, V_i^{d} obeys:

$$C_m \frac{dV_i^{\mathrm{d}}}{dt} = -I_{leak,i}^{\mathrm{d}} - I_{NaP,i} - I_{Ks,i} - I_{KA,i} - \frac{g_c}{1-p}(V_i^{\mathrm{d}} - V_i^{\mathrm{s}}) - I_{syn}, \qquad (9)$$

where C_m is the specific membrane capacitance which is taken to be 1 μF/cm^2 for both the dendrite and the soma for all cells and p =0.2, g_c =0.025 determining the electrotonic structure of the neuron. A small (6 neuron) version of the simulation program can be found at

`http://www.bio.brandeis.edu/lismanlab/integrator`. The program can be run using the differential equation solver package XPP by Bard Ermentrout (`http://www.math.pitt.edu/~bard/xpp`).

Membrane currents

The leak conductance, g_{leak}^{s}, is 0.3 mS/cm^2 for the soma and a value uniformly distributed on the interval (0.2,0.9) mS/cm^2 for the dendrite. $E_{leak} = -75$ mV was the leak reversal potential for both the compartments. The spiking currents were the same as in the circuit-based bistability model above except that the sodium current was taken to activate instantaneously $\left[m_\infty(V) = \frac{\alpha_m(V)}{\alpha_m(V)+\beta_m(V)}\right]$, with the activation variables given as $\alpha_m(V) = -0.1(V{+}43)/[\exp(-(V{+}43)/10){-}1]$, $\beta_m(V) = 4\exp(-[V{+}68]/18)$; $\alpha_h(V) = 0.07\exp(-[V{+}66]/20)$ and $\beta_h(V) = 1/[\exp(-\{V{+}36\}/10){+}1]$, with $\phi_h = 2.5$ and with $g_{Na} = 35$ mS/cm^2.The delayed rectifier potassium current, $I_{KDr} = g_K n^4(V^{\mathrm{s}} - E_K)$, where $g_K = 9$ mS/cm^2 and potassium reversal potential, $E_K = -90$ mV with $\alpha_n(V) = -0.007(V+42)/[\exp(-(V+42)/10) - 1]$, $\beta_n(V) = 0.14\exp(-[V+52]/80)$, with $\phi_n = 2.5$. and $g_K = 9$ mS/cm^2.

In the dendrite, the persistent sodium current, $I_{NaP} = g_{NaP} r_\infty^3(V)(V - E_{Na})$, with $r_\infty(V) = 1/(1+\exp(-(V+58)/5))$ and g_{NaP} =0.25 mS/cm^2. The two potassium currents were $I_{Ks} = g_{Ks} q(V - E_K)$, with $q_\infty(V) = 1/(1+\exp(-(V+50)/2))$ and $\tau_q(V) = 200/(\exp(-(V+60)/10)+\exp((V+60)/10))$ and $g_{KS} = 0.1$ mS/cm^2; and $I_{KA} = g_{KA} a_\infty^3(V) b(V - E_K)$, with $a_\infty(V) =$

$1/(1+\exp(-(V+44)/6))$, $b_\infty(V) = 1/(1+\exp((V+56)/15))$ and $\tau_b(V) = 2.5(1+\exp((V+60)/30))$ and $g_{KA} = 10$ mS/cm^2.

The synaptic current into each neuron is due to the recurrent connections and the external (vestibular) inputs:

$$I_{syn,i} = I_{NMDA,i} + I_{AMPA,i} + I_{ext} + I_{noise}, \tag{10}$$

with $I_{NMDA,i}$ as above with $g_{NMDA,i} = 0.1$ mS/cm^2. For the randomized version of the network, $g_{NMDA,i}$ was chosen from a Gaussian distribution with a mean of 0.1 mS/cm^2 and a standard deviation of 0.0125 mS/cm^2, g_{NaP} was chosen from a Gaussian distribution with a mean of 0.25 mS/cm^2 with a standard deviation of 0.025 mS/cm^2, the leak conductances were chosen from a uniform distribution (0.2,0.9) mS/cm^2 and g_{Ks} was chosen from a mean of 0.1 mS/cm^2 with a standard deviation of 0.01 mS/cm^2.

The AMPA synaptic current is:

$$I_{AMPA,i} = g_{AMPA} \sum_j x_{AMPA,j}(V_i^{\mathrm{d}} - E_{syn}), \tag{11}$$

with $g_{AMPA} = 0.02$ mS/cm^2 and $E_{syn} = 0$ mV for both AMPA and NMDA-type synapses. The AMPA gating variable obeys first-order kinetics as:

$$\frac{dx_{AMPA,j}}{dt} = 40F(V_j^{\mathrm{s}})(1 - x_{AMPA,j}) - x_{AMPA,j}, \tag{12}$$

where the gating function, $F(V_j^{\mathrm{s}}) = 1/(1+\exp[-(V_j^{\mathrm{s}}+10)/2])$. The network was all-to-all connected, without any self-excitation (autapses). Each neuron also received external input from excitatory and inhibitory burst neurons, $I_{ext} = g_{ex}x_{ex}(V - E_{syn}^{ex}) + g_{inh}x_{inh}(V - E_{syn}^{inh})$, with $g_{ex} = 0.2$ mS/cm^2 and $g_{inh} = 0.3$ mS/cm^2. A noise current, I_{noise}, is added to each dendrite, modeled as $g_{noise}x_{noise}(V - E_{syn}^{ex})$, with x_{noise} represents conductance kicks from a Poisson spike-train of 500 Hz and $g_{noise} = 0.01$ mS/cm^2. The differential equations were integrated using a fourth-order Runge-Kutta method modified for stochastic input with a time step of 0.025 ms.

Since the recurrent connections in our model are moderately saturating, presynaptic firing rates of about 10-15 Hz (for our value of the NMDA time constant) drive s near 1. Thus, each active presynaptic cell in the network contributes an order unity factor to the total NMDA input to a postsynaptic cell. Hence, s can be used a measure of the total network activity. Without active currents in the dendrites, the bistable region of each neuron is limited to a small range of network activity. The addition of active currents in the dendrites extends this region of bistability, such that each neuron is bistable over a larger range of s and consequently a larger region of total network activity. The robustness of the nework stems from this expanded range of bistability. Note, however, the intrinsic currents alone do not lead to bistability without NMDA input.

Network inputs

The integrator network receives feedforward input from excitatory and inhibitory burst neurons which are modeled by fast (AMPA-like) synapses with synaptic reversal potentials $E^e_{syn} = 0$ mV for excitatory burst inputs and $E^i_{syn} = -80$ mV for inhibitory burst inputs. The burst neurons themselves are modeled by single compartment neurons with only the spiking currents above and driven by somatic current pulses of 30 ms width to mimic on- and off-direction saccades. The duration and magnitude of injected currents controls the duration of the input bursts. The input burst duration, as well as g_{ex} and g_{inh} control the overall gain of the network, i.e. the number of units that are turned on and off with each burst.